

2021 ELECTIVE MATHEMATICS PAPER 2 SOLUTION

1.

The polynomial $f(x) = 2x^3 + px^2 + qx - 5$ has $(x-1)$ as a factor and a remainder of 27 when divided by $(x + 2)$, where p and q are constants. Find the values of p and q .

SOLUTION

$$f(x) = 2x^3 + px^2 + qx - 5$$

$$f(1) = 2(1)^3 + p(1)^2 + q(1) - 5$$

$$2 + p + q - 5;$$

$$p + q = 3 \text{ -- (i)}$$

$$f(-2) = 27$$

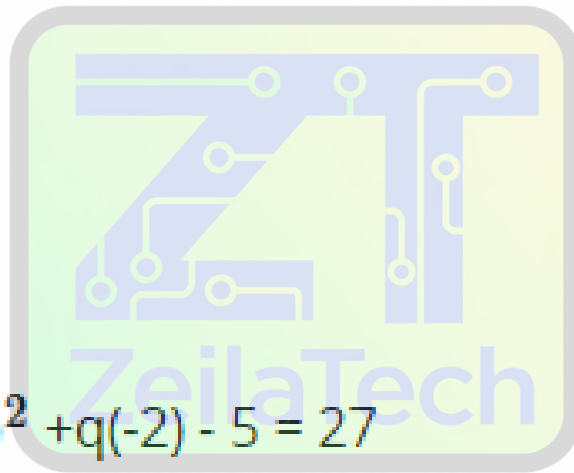
$$\text{i.e., } 2(-2)^3 + p(-2)^2 + q(-2) - 5 = 27$$

$$-16 + 4p - 2q - 5 = 27$$

$$2p - q = 24 \text{ --- (ii)}$$

Solving (i) and (ii) Simultaneously

$$P = 9, q = -6$$



Smart Learning Tools

2.

Evaluate: $9 \int_1^9 \frac{x(2x-3)}{\sqrt{x}} dx$

SOLUTION

$$9 \int_1^9 \frac{x(2x-3)}{\sqrt{x}} dx = \frac{2x^2-3x}{x^{1/2}}$$

$$= x^{-1/2} (2x^2 - 3x)$$

$$= 2x^{3/2} - 3x^{1/2}$$

$$= \frac{2x^{3/2}}{3/2+1} - \frac{3x^{1/2}}{1/2+1}$$

$$= \frac{2x^{3/2}}{5/2} - \frac{3x^{1/2}}{3/2}$$

$$= 9 \int_1^9 \left[\frac{4x^{3/2}}{5} - \frac{6x^{1/2}}{3} \right]$$

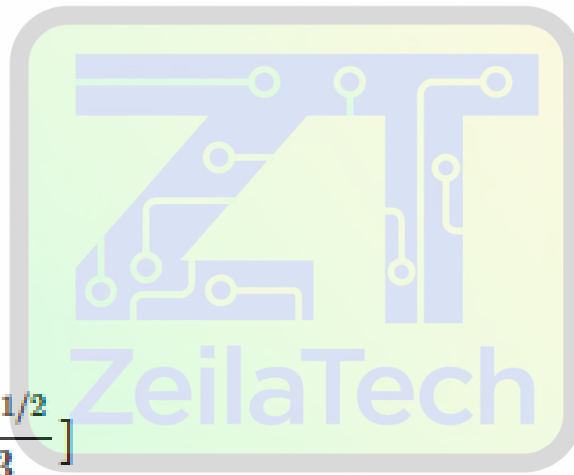
$$\left[\frac{4(9)^{3/2}}{5} - \frac{6(9)^{1/2}}{3} \right] - \left[\frac{4(1)^{3/2}}{5} - \frac{6(1)^{1/2}}{3} \right]$$

$$= \left[\frac{4(27)}{5} - \frac{6(3)}{3} \right] - \left[\frac{4}{5} - 2 \right]$$

$$= \left[\frac{108}{5} - 2 \right] - \left[\frac{6}{5} \right]$$

$$= \frac{78}{5} + \frac{6}{5} = \frac{84}{5}$$

$$16^{4/5}$$



Smart Learning Tools

3.

Given that $(p + 1/2\sqrt{3})(1 - \sqrt{3})^2 = 3 - \sqrt{3}$,

find x the value of p.

SOLUTION

$$(p + 1/2\sqrt{3})(1 - \sqrt{3})^2 = 3 - \sqrt{3}$$

$$(1 - \sqrt{3})^2 = 4 - 2\sqrt{3}$$

$$(p + 1/2\sqrt{3})(4 - 2\sqrt{3}) = 3 - \sqrt{3}$$

$$4p - 2p\sqrt{3} - 2\sqrt{3} - 3 = 3 - \sqrt{3}$$

$$4p - 2p\sqrt{3} = 3 - \sqrt{3} + 3 + 2\sqrt{3}$$

$$= 2p(2 - \sqrt{3}) = 6 + \sqrt{3}$$

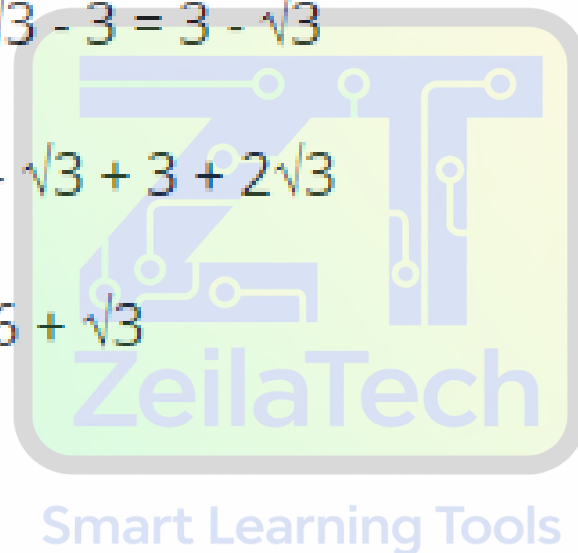
$$2p = \frac{6 + \sqrt{3}}{2 - \sqrt{3}}$$

$$p = \frac{2(6 + \sqrt{3})}{2 - \sqrt{3}}$$

$$= \frac{12 + 2\sqrt{3}}{2 - \sqrt{3}}$$

$$= \frac{(12 + 2\sqrt{3})(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}$$

$$: 30 + 16\sqrt{3}$$



4.

${}^{5y}C_2 = 190$, find the value of y

SOLUTION

$${}^{5y}C_2 = 190$$

$$\frac{5y(5y-1)(5y-2)!}{(5y-2)!(2)!} = 190$$

$$\frac{5y(5y-1)}{2} = 190$$

$$25y^2 - 5y = 380$$

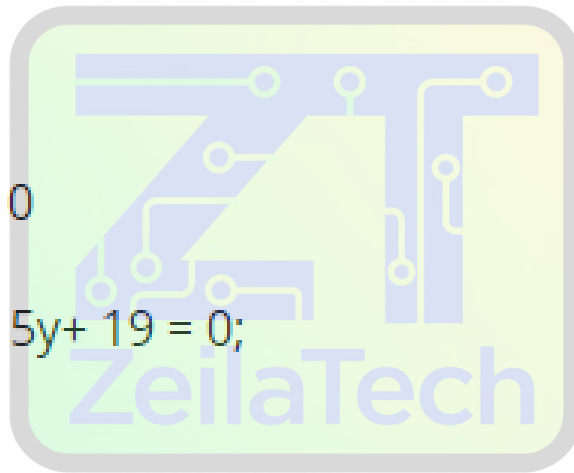
$$5y^2 - 5y - 76 = 0$$

$$= (y-4)(5y+19) = 0$$

Either $y - 4 = 0$ or $5y + 19 = 0$;

$$y = 4 \text{ or } -19/5$$

$$\therefore y = 4$$



Smart Learning Tools

5.

The table shows the distribution of monthly income (in thousands of naira) of workers in a factory

Monthly Income (GHC1000)	135-139	140-149	150-154	155-164	165-169
Number of workers	20	42	28	38	22

(a) Draw a histogram for the distribution.

(b) Use your graph to estimate the mode of the distribution.

SOLUTION

Monthly Income	Number of Workers	Class Boundaries
135-139	20	134.5 - 139.5
140-149	42	139.5 - 149.5
150-154	28	149.5 - 154.5
155-164	38	154.5 - 164.5
165-169	22	164.5 - 169.5

(a)

(b) Modal class: This is the class boundary with the highest frequency.

In this case, it is 139.5 and 149.5.

From the histogram, the modal income is $3 + 139.5$

$= 142.5$

6.

A bag contains 24 mangoes out of which six are bad. If 6 mangoes are selected randomly from the bag with replacement, find the probability that not more than 3 are bad.

SOLUTION

Let. X = Prob. that a good mango is selected $= 18/24 = 3/4$

Y Prob. that a bad mango is selected $= 6/24 = 1/4$

Using the binomial probability distribution, we have:

$$(X+Y)^6 = X^6 + {}^6C_1 X^5 Y + {}^6C_2 X^4 Y^2 + {}^6C_3 X^3 Y^3 + {}^6C_4 X^2 Y^4 + {}^6C_5 X Y^5 + {}^6C_6 Y^6$$

Probability that not more than 3 are bad is

$$= {}^6C_1 X^5 Y + {}^6C_2 X^4 Y^2 + {}^6C_3 X^3 Y^3$$

$$= 6(3/4)^5 (1/4) + 15(3/4)^4 (1/4)^2 + 20(3/4)^3 (1/4)^3$$

$$= 6(243/1024)(1/4) + 15(81/256)(1/16) + 20(27/64)(1/64)$$

$$= 0.36 + 0.30 + 0.13$$

$$\approx 0.79$$

7.

(a) The speed of a moving bus reduced from 45m/s to 5m/s with a uniform retardation of 10m/s². Calculate the distance covered.

(b) A bucket full of water with mass 16kg is pulled out of a well with a light inextensible rope. Find its acceleration when the tension in the rope is 240N. [Take g= 10m/s²]

SOLUTION

(a) Using $v^2 = u^2 + 2as$ ($v=5\text{m/s}$, $u=45\text{m/s}$)

$v^2 = u^2 - 2as$ (For retardation)

$$5^2 = 45^2 - 2(10)s$$

$$5^2 - 45^2 = 20s;$$

$$50(-40) = -20s$$

$$-2000 = -20s;$$

$$s=100\text{m}$$

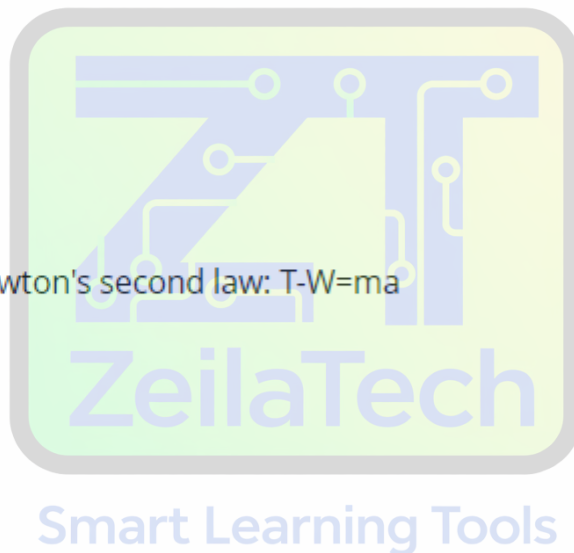
(b) Net force is T- W By Newton's second law: $T-W=ma$

$$240 - 26 \times 10 = 16a$$

$$240 - 260 = 16a$$

$$-20 = 16a$$

$$a = -1.25\text{m/s}^2$$



8.

Given that $x = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ and $y = \begin{pmatrix} -9 \\ 15 \end{pmatrix}$ calculate, correct to the nearest degree, the angle between the vectors.

SOLUTION

$$x = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \text{ and } y = \begin{pmatrix} -9 \\ 15 \end{pmatrix}$$

Changing x and y to the form xi+yj

we have $x = -4i + 3j$ and $y = -9i - 15j$

$$\text{using } \cos \theta = \frac{xy}{|x||y|}$$

where θ is the angle between x and y

$$xy = (-4i+3j)(-9i-15j)$$

$$-36 + 60 \times 0 - 27 \times 0 - 45 = -81$$

$$|x| = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$|y| = \sqrt{(-9)^2 + (-15)^2} = \sqrt{81 + 225} = \sqrt{306}$$

$$\cos \theta = \frac{-81}{5\sqrt{306}}$$

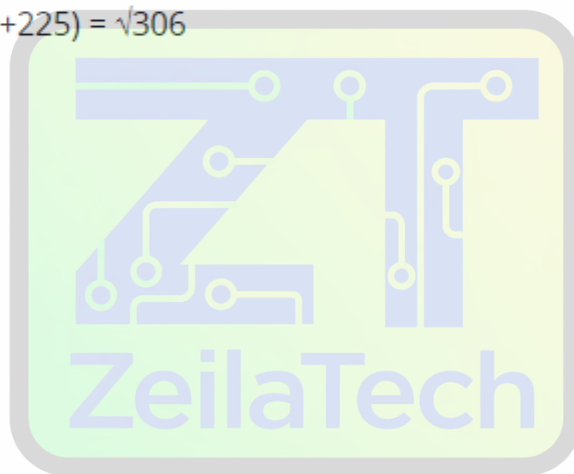
$$= \frac{-9\sqrt{306}}{170}$$

$$\frac{17.49 \times 9}{170}$$

$$\theta = \cos^{-1}(-0.1029);$$

$$\theta = 95.91^\circ$$

$$\theta = 96^\circ$$



Smart Learning Tools

9.

(a) A jogger is training for 15km charity race. He starts with a run of 500 metres, then he increases the distance he runs daily by 250 metres.

(i) How many days will it take the jogger to reach a distance of 15km in training?

(ii) Calculate the total distance he would have run in the training.

(b) The second term of a Geometric Progression (GP) is -3. If its sum to infinity is $25/2$, find its common ratios.

SOLUTION

(a)1) The sequence is an A.P: 500, 750, 1000 ..

with $a = 500$ and $d=250$; $T_n=15000$

Using $T_n = a + (n - 1)d$

$$15000 = 500 + (n - 1) \times 250$$

$$15000 = 500 + 250n - 250$$

$$250n = 14500 ; n = \frac{14500}{250}$$

$$= 58$$

The jogger will reach a distance of 15km in 58 days.

(ii) Finding total distance he would have run in the training

Using $S_n = n/2 [2a + (n - 1)d]$ $n= 58$, $d=250$, $a = 500$

$$= 58/2 [2 \times 500 + (58 - 1) \times 250]$$

$$= 29 [1000 + (57 \times 250)]$$

$$= 29 (1000 + 14250),$$

$$= 29 \times 15250 = 442250$$

(b) 2nd term $\Rightarrow ar = -3$; $a = -3/r$

$$S_{\infty} = \frac{a}{1-r} = 25/2$$

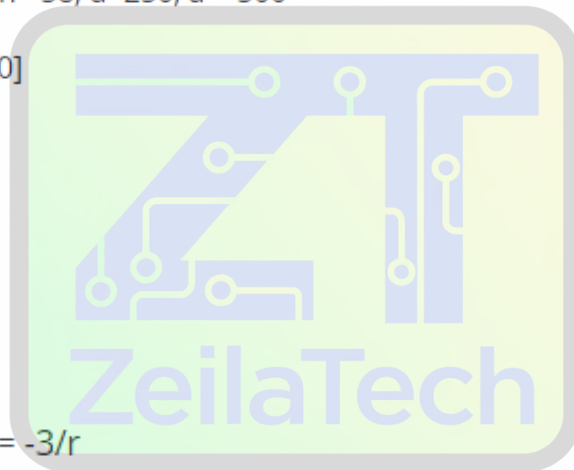
$$\frac{-3}{r} \times \frac{1}{1-r} = 25/2;$$

$$25r - 25r^2 = -6;$$

$$25r^2 - 25r - 6 = 0$$

$$(5r + 1) (5r - 6) = 0$$

$$r = -1/5 \text{ and } 6/5$$



Smart Learning Tools

10.

P and Q are two linear transformations in the X-Y plane defined by

$$P: (x, y) \rightarrow (-3x + 6y, 4x + y) \text{ and}$$

$$Q: (x, y) \rightarrow (2x - 3y, -4x - 6y).$$

(a) Write down the matrices of P and Q. (b) What is the image of $(-2, -3)$ under the transformation Q?

(c) Obtain a single transformation representing the transformation Q followed by P.

(d) Find the image of $(1, 4)$ when transformed by Q followed by P.

(e) Find the image P^{-1} of the point $(-\sqrt{2}, 2\sqrt{2})$ under an anticlockwise rotation of 225° about the origin.

SOLUTION

$$P: (x, y) \rightarrow (2x + 3y, 3x - y)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -3x & 6y \\ 4x & +y \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 6 \\ 4 & +1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

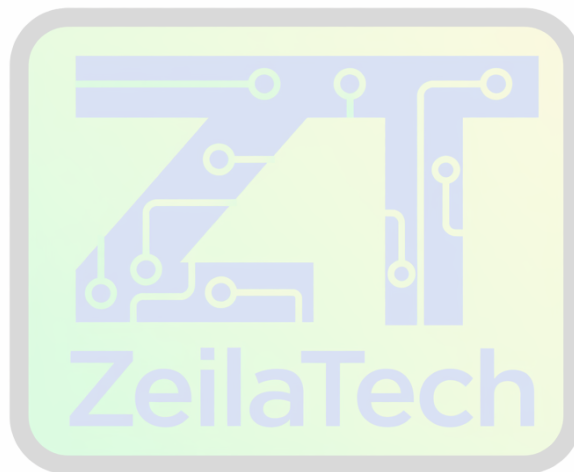
$$P = \begin{pmatrix} -3 & 6 \\ 4 & +1 \end{pmatrix}$$

$$Q: (x, y) \rightarrow (2x - 3y, -4x - 6y)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2x & -3y \\ -4x & -6y \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -3 \\ -4 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$Q = \begin{pmatrix} 2 & -3 \\ -4 & -6 \end{pmatrix}$$



Smart Learning Tools

$$\begin{aligned}
 \text{(b)} & \begin{pmatrix} 2 & -3 \\ -4 & -6 \end{pmatrix} \begin{pmatrix} -2 \\ -3 \end{pmatrix} \\
 &= \begin{pmatrix} 2[-2] & -3[-3] \\ -4[-2] & -6[-3] \end{pmatrix} \\
 &= \begin{pmatrix} -4 & +9 \\ 8 & 18 \end{pmatrix} \\
 &= \begin{pmatrix} 5 \\ 26 \end{pmatrix}
 \end{aligned}$$

: The image of (-2,-3) is (5,26)

$$\text{(c)} PQ = \begin{pmatrix} -3 & 6 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ -4 & -6 \end{pmatrix}$$

$$\begin{aligned}
 & \begin{pmatrix} -3[2] + 6[-4] & -3[-3] + 6[-6] \\ 4[2] + 1[-4] & 4[-3] + 1[-6] \end{pmatrix} \\
 &= \begin{pmatrix} -6 - 24 & 9 - 36 \\ 8 - 4 & -12 - 6 \end{pmatrix} = \begin{pmatrix} -30 & -27 \\ 4 & -18 \end{pmatrix}
 \end{aligned}$$

$$= PQ: (x,y) \rightarrow (-30x - 27y, 4x - 18y)$$

$$\text{(d)} \begin{pmatrix} -30 & -27 \\ 4 & -18 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} -30[1] + [-27][4] \\ 4[1] + [-18][4] \end{pmatrix}$$

$$= \begin{pmatrix} -30 & -108 \\ 4 & -72 \end{pmatrix} = \begin{pmatrix} -138 \\ -68 \end{pmatrix}$$

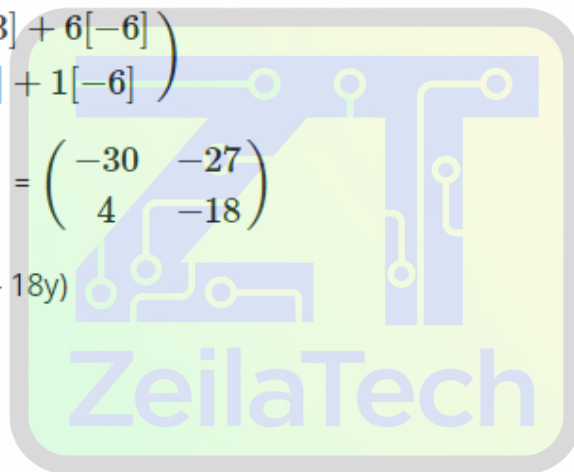
: Image of the point (1,4) is (-138,-68)

$$\text{(e)} T = \begin{pmatrix} \cos 225^\circ & -\sin 225^\circ \\ \sin 225^\circ & \cos 225^\circ \end{pmatrix}$$

$$= \begin{pmatrix} -\sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & -\sqrt{2}/2 \end{pmatrix} \begin{pmatrix} -\sqrt{2} \\ 2\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & +2 \\ 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

: The image of P^1 is (3,-1)



Smart Learning Tools

11.

(a) Find the equation of the normal to the curve $y = (x^2 - x + 1)(x - 2)$ at the point where the curve cuts the X - axis.

(b) The coordinates of the points P, Q and R are $(-1, 2)$, $(5, 1)$ and $(3, -4)$ respectively. Find the equation of the line joining Q and the midpoint of line PR.

SOLUTION

$$(a) \text{ From } y = (x^2 - x + 1)(x - 2) = x^3 - 3x^2 + 3x - 2$$

$$= x^3 - 3x^2 + 3x - 2$$

$$dy/dx = 3x^2 - 6x + 3$$

$$= x^2 - 2x + 1$$

$$(x - 1) = 0 \text{ twice ; } x = 1$$

$$y = (1^2 - 1 + 1)(1 - 2); y = -1$$

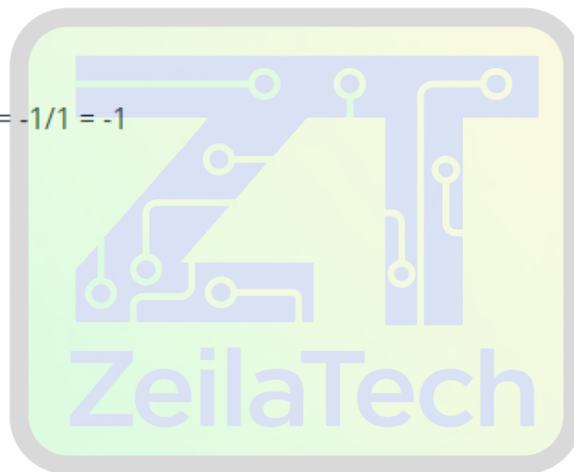
$$\text{Gradient } m_1 \text{ of tangent} = y/x = -1/1 = -1$$

$$\text{Using } m_1 m_2 = -1$$

$$\text{Gradient, } m_2 \text{ of normal} = 1$$

$$\text{Using } y - y_1 = m_2(x - x_1)$$

$$y + 1 = x - 1; x - y - 2 = 0$$



$$x_1, y_1 = (-1, 2), , x_2, y_2, = (3, -4) \text{ Smart Learning Tools}$$

$$\text{Mid point of PR} = \frac{3-1, -4+2}{2, 2}$$

$$= (1, -1)$$

$$\text{Using } \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$\frac{1 - (-1)}{5 - 1} = \frac{y - 1}{x - 5}$$

$$\frac{2}{4} = \frac{y - 1}{x - 5}$$

$$2(y - 1) = x - 5; 2y - 2 = x - 5$$

$$2y - x = -3$$

$$x - 2y - 3 = 0$$

12.

A box contains 5 red, 7 blue and 4 green identical bulbs. Two bulbs are picked at random from the box without replacement.

Calculate the probability of picking: (a) same color of bulbs; (b) different color of bulbs (c) at least one red bulb.

. Total bulbs: $n(\text{Red})=5$, $n(\text{B}) = 7$, $n(\text{G}) = 4$

$$(5+7+4) = 16$$

$$p(\text{R})= 5/16, p(\text{B}) = 7/16, p(\text{G}) = 4/16$$

(a) $p(\text{All same colour of bulbs}) = p(\text{RR}) \text{ Or } p(\text{BB}) \text{ or } p(\text{GG})$

$$\frac{5}{16} * \frac{4}{15} + \frac{7}{16} * \frac{6}{15} + \frac{4}{16} * \frac{3}{15}$$

$$= \frac{1}{12} + \frac{7}{40} + \frac{1}{20}$$

$$= \frac{10+21+6}{120} = \frac{37}{120}$$

(b) All different colors = $1 - p(\text{All the same colour}) =$

$$1 - \frac{37}{120} = \frac{83}{120}$$

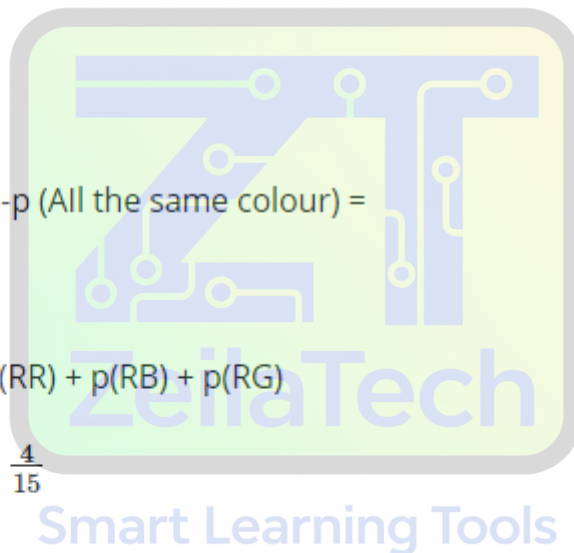
(c) $p(\text{At least one red}) = p(\text{RR}) + p(\text{RB}) + p(\text{RG})$

$$\frac{5}{16} * \frac{4}{15} + \frac{5}{16} * \frac{7}{15} + \frac{5}{16} * \frac{4}{15}$$

$$= \frac{1}{12} + \frac{7}{48} + \frac{1}{12}$$

$$= \frac{8+7}{48} = \frac{15}{48}$$

$$= \frac{5}{16}$$



13.

The table shows the frequency distribution of heights (in cm) of pupils in a certain school.

Heights	100-109	110-119	120-129	130-139	140-149	150-159	160-169
Frequency	27	58	130	105	50	25	5

- (a) (i) Construct a cumulative frequency table.
- (ii) Use the table to draw a cumulative frequency curve.
- (b) Using the curve, estimate the:
- (i) median height;
- (ii) inter quartile range
- (iii) percentage of students whose heights are most 130cm.

SOLUTION

Height (in cm)	Frequency
Not exceeding 99.5	0
Not exceeding 109.5	27
Not exceeding 119.5	85
Not exceeding 129.5	215
Not exceeding 139.5	320
Not exceeding 149.5	370
Not exceeding 159.5	395
Not exceeding 169.5	400

(ii)

bi) Median $1/2$ of $N = 1/2 \times 400 = 200$

Cumulative frequency level From the curve, it is 118.5cm

(ii) Inter quartile range = upper quartile - lower quartile

= $3/4$ of N

= $3/4 \times 400 = 300$

Cumulative frequency level From the curve, it is 126.5cm

lower quartile = $1/4$ of N

= $1/4 \times 400 = 100$

Cumulative frequency level From the curve, it is 111.0cm

Inter quartile range = $126.5\text{cm} - 111.0\text{cm} = 15.5\text{cm}$

(ii) From the curve, students whose heights are at most 130cm are 240.

Percentage of students whose heights are at most 130cm = $\frac{240}{400} \times 100$

= 60%

14.

The position vectors of P, Q and R with respect to the origin are $(4i-5j)$, $(i+3j)$ and $(-5i+2j)$ respectively. If PQRM is a parallelogram, find:

(a) the coordinates of M;

(b) the acute angle between \overline{PM} and \overline{PQ} , correct to the nearest degree.

SOLUTION

$$\overline{PQ} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$$

$$\overline{MR} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5-x \\ 2-y \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 8 \end{pmatrix} = \begin{pmatrix} -5-x \\ 2-y \end{pmatrix}$$

$$\Rightarrow -5-x = -3; x = -2$$

$$\Rightarrow 2-y = 8; y = -6$$

The coordinate of M $(-2, -6)$

$$(b) \overline{PM} = \overline{OM} - \overline{OP}$$

$$= \begin{pmatrix} -2 \\ -6 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \end{pmatrix}$$

$$\overline{PQ} = \overline{OQ} - \overline{OP}$$

$$= \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$$

$$|\overline{PM}| = \sqrt{(-6)^2 + (-1)^2} = \sqrt{37}$$

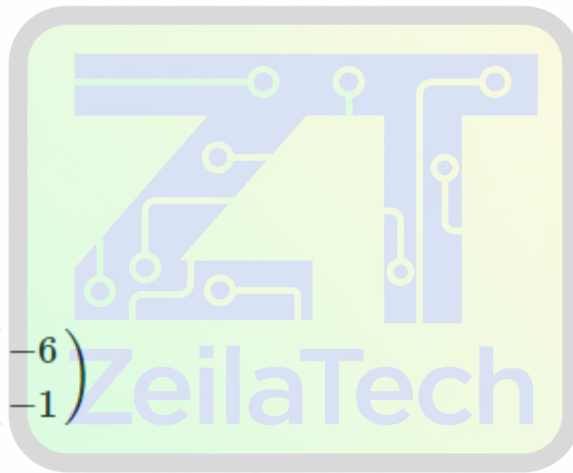
$$|\overline{PQ}| = \sqrt{(-3)^2 + 8^2} = \sqrt{73}$$

$$\cos \theta = \frac{\overline{PM} \cdot \overline{PQ}}{|\overline{PM}| |\overline{PQ}|}$$

$$\cos \theta = \frac{[-6i-j] \cdot [-3i+8j]}{\sqrt{37} \cdot \sqrt{73}} = \frac{10}{\sqrt{2710}}$$

$$\Rightarrow \theta = \cos^{-1} \frac{10}{\sqrt{2710}}$$

$$\therefore \theta = 78.91^\circ \approx 79^\circ$$



Smart Learning Tools

15.

(a) A girl threw a stone horizontally with a velocity of 30m/s from the top of a cliff 50m high. How far from the foot of the cliff does the stone strike the ground? [Take $g= 10\text{m/s}^2$]

SOLUTION

(a) Time taken to reach the ground

$$\text{Using } S = ut + \frac{1}{2}gt^2$$

($u= 0$ for a body falling from rest)

$$50 = 0 \times t + \frac{1}{2} \times 10t^2$$

$$5t^2 = 50;$$

$$t^2 = 50/5 = 10$$

$$t = \sqrt{10} = 3.16$$

Distance when it strikes the ground This journey is independent of gravity

$$d = vt = 30 \times 3.16 = 94.8\text{m}$$

(b) A body A, of mass 2kg is held in equilibrium by means of two strings AP and AR. AP is inclined at 56° to the upward vertical and AR is horizontal.

Find the tensions T_1 , and T_2 , in the strings [Take $g= 10\text{ms}^2$]

SOLUTION

(b) Using Lami's rule,

$$\begin{aligned} \frac{20}{\sin 34} &= \frac{T_1}{\sin 90} = \frac{T_2}{\sin 56} \\ &= T_1 \frac{20 \cdot \sin 90}{\sin 34} = T_2 \frac{20 \cdot \sin 56}{\sin 34} \end{aligned}$$

$$T_1 = 35.79\text{N and } T_2 = 29.65\text{N}$$