# 2021 ELECTIVE MATHEMATICS PAPER 2 SOLUTION

1.

The polynomial $f(x) = 2x^3 + px + qx - 5$ has $(x-1)$ as a factor and a remainder of 27 when divided by $(x + 2)$, where p and q are constants. Find the values of p and q.

SOLUTION

$f(x) = 2xp^3 + px^2 + qx - 5$

$f(1) = 2(1)^3 + p(1)^2 + q(1) - 5$

$2 + p + q - 5;$

$p + q = 3$ -- (i)

$f(-2) = 27$

i.e, $2(-2)^3 + p(-2)^2 + q(-2) - 5 = 27$

$-16 + 4p - 2q - 5 = 27$

$2p - q = 24$--- (ii)

Solving(i) and (ii) Simultaneously

$P = 9, q = -6$

2.

Evaluate: $^{9}\int_{1} \frac{x(2x-3)}{\sqrt{x}}$ dx

SOLUTION

$$^{9}\int_{1} \frac{x(2x-3)}{\sqrt{x}} \, dx = \frac{2x^2-3x)}{x^{1/2}}$$

$$= x^{-1/2} (2x^2 - 3x)$$

$$= 2x^{3/2} - 3x^{1/2}$$

$$= \frac{2x^{3/2}}{3/2+1} - \frac{3x^{1/2}}{1/2+1}$$

$$= \frac{2x^{3/2}}{5/2} - \frac{3x^{1/2}}{3/2}$$

$$= {}^{9}\int_{1} [\frac{4x^{3/2}}{5} - \frac{6x^{1/2}}{3}]$$

$$[ \frac{4(9)^{3/2}}{5} - \frac{6(9)1/2}{3} ] - [ \frac{4(9)^{1/2}}{5} - \frac{6(1)^{1/2}}{3} ]$$

$$= [ \frac{4(27)}{5} - \frac{6(3)}{3} - [ \frac{4}{5} - 2 ]$$

$$= [ \frac{108}{5} - 2 ] - [ \frac{6}{5} ]$$

$$= \frac{78}{5} + \frac{6}{5} = \frac{84}{5}$$

$$16^{4/5}$$

3.

Given that $(p + 1/2\sqrt{3})(1 - \sqrt{3})^2 = 3 - \sqrt{3}$,

find x the value of p.

SOLUTION

$$(p + 1/2\sqrt{3})(1 - \sqrt{3})^2 = 3 - \sqrt{3}$$

$$(1 - \sqrt{3})^2 = 4 - 2\sqrt{3}$$

$$(p + 1/2\sqrt{3})\ (4 - 2\sqrt{3}) = 3 - \sqrt{3}$$

$$4p - 2p\sqrt{3}\ \ - 2\sqrt{3} - 3 = 3 - \sqrt{3}$$

$$4p - 2p\sqrt{3} = 3 - \sqrt{3} + 3 + 2\sqrt{3}$$

$$= 2p(2 - \sqrt{3}) = 6 + \sqrt{3}$$

$$2p = \frac{6+\sqrt{3}}{2-\sqrt{3}}$$

$$p = \frac{2(6+\sqrt{3})}{2-\sqrt{3}}$$

$$= \frac{12+2\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{(12+2\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$: 30 + 16\sqrt{3}$$

4.

${}^{5y}C_2 = 190$, find the value of y

SOLUTION

$${}^{5y}C_2 = 190$$

$$\frac{5y(5y-1)(5y-2)!}{(5y-2)!(2)!} = 190$$

$$\frac{5y(5y-1)}{2} = 190$$

$$25y^2 - 5y = 380$$

$$5y^2 - 5y - 76 = 0$$

$$= (y-4)(5y+19) = 0$$

Either $y - 4 = 0$ or $5y + 19 = 0$;

$y = 4$ or $-19/5$

: $y = 4$



5.

The table shows the distribution of monthly income (in thousands of naira) of workers in a factory

| Monthly Income (GHC1000) | 135-139 | 140-149 | 150-154 | 155-164 | 165-169 |
|---|---|---|---|---|---|
| Number of workers | 20 | 42 | 28 | 38 | 22 |

(a) Draw a histogram for the distribution.

(b) Use your graph to estimate the mode of the distribution.

SOLUTION

| Monthly Income | Number of Workers | Class Boundaries |
| --- | --- | --- |
| 135-139 | 20 | 134.5 - 139.5 |
| 140-149 | 42 | 139.5 - 149.5 |
| 150-154 | 28 | 149.5 - 154.5 |
| 155-164 | 38 | 154.5 - 164.5 |
| 165-169 | 22 | 164.5 - 169.5 |

(a)

(b) Modal class: This is the class boundary with the highest frequency.

In this case, it is 139.5 and 149.5.

From the histogram, the modal income is 3 +139.5

= 142.5

6.

A bag contains 24 mangoes out of which six are bad. If 6 mangoes are selected randomly from the bag with replacement, find the probability that not more than 3 are bad.

SOLUTION

Let. X = Prob. that a good mango is selected = 18/24= 3/4

Y Prob. that a bad mango is selected = 6/24 1/4

Using the binomial probability distribution, we have:

$(X+Y)^6 = X^6 + {}^6C_1X^5Y + {}^6C_2X^4Y^2 + {}^6C_3X^3Y^3 + {}^6C_4X^2Y^4 + {}^6C_5XY^5 + {}^6C_6Y^6$

Probability that not more than 3 are bad is

$= {}^6C_1X^5Y + {}^6C_2X^4Y^2 + {}^6C_3X^3Y^3$

$= 6(3/4)^5 (1/4) + 15(3/4)^4 (1/4)^2 + 20(3/4)^3 (1/4)^3$

= 6(243/1024)(1/4) + 15(81/256)(1/16) + 20(27/64)(1/64)

= 0.36 + 0.30 + 0.13

≈ 0.79

7.

(a) The speed of a moving bus reduced from 45m/s to 5m/s with a uniform retardation of $10m/s^2$. Calculate the distance covered.

(b) A bucket full of water with mass 16kg is pulled out of a well with a light inextensible rope. Find its acceleration when the tension in the rope is 240N. [Take g= $10m/s^2$]

SOLUTION

(a) Using $v^2 = u^2 + 2as$ (v=5m/s, u=45m/s)

$v^2 = u^2 - 2as$ (For retardation)

$5^2 = 45^2 - 2(10)s$

$5^2 - 45^2 = 20s;$

$50(-40) = -20s$

$-2000 = -20s;$

s=100m

(b) Net force is T- W By Newton's second law: T-W=ma

240 - 26x10 = 16a

240 - 160 = 16a

80 16a

a = 5m/s



8.

Given that x = $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ and y = $\begin{pmatrix} -9 \\ 15 \end{pmatrix}$ calculate, correct to the nearest degree, the angle between the vectors.

SOLUTION

x = $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ and y= $\begin{pmatrix} -9 \\ 15 \end{pmatrix}$

Changing x and y to the form xi+yj

we have x= -4i + 3j and y = -9i - 15j

using cosØ = $\frac{xy}{|x|ly|}$

where Ø is the angle between x and y

xy = (-4i+3j) (-9i - 15j)

-36 + 60 x 0 - 27 x 0- 45 = -81

|x| = √($4^2$ +$3^2$) =√(16 +9) = √25 = 5

|y| = √$(-9)^2$ + $(-15)^2$ = √(81 +225) = √306

cosØ =$\frac{-81}{5\sqrt{306}}$

= $\frac{-9\sqrt{306}}{170}$

$\frac{17.49*9}{170}$

Ø = $\cos^{-1}$(-0.1029);

Ø = 95.91°

Ø = 96°



9.

(a) A jogger is training for 15km charity race. He starts with a run of 500 metres, then he increases the distance he runs daily by 250 metres.

(i) How many days will it take the jogger to reach a distance of 15km in training?

(ii) Calculate the total distance he would have run in the training.

(b) The second term of a Geometric Progression (GP) is -3. If its sum to infinity is 25/2, find its common ratios.

SOLUTION

(a)1) The sequence is an A.P: 500, 750, 1000 ..

with a = 500 and d=250; Tn=15000

Using $T_n$ = a + (n - 1)d

15000 = 500+ (n - 1) x 250

15000 = 500 +250n - 250

250n = 14500 ; n = $\frac{14500}{250}$

= 58

The jogger will reach a distance of 15km in 58 days.

(ii) Finding total distance he would have run in the training

Using $S_n$ =n/2 [2a + (n - 1)d] n= 58, d=250, a = 500

= 58/2 [2 x 500 + (58 -1) x 250]

= 29[ 1000 +(57 x 250)]

= 29 (1000+ 14250),

= 29 x 15250 = 442250

(b) 2nd term => ar = -3; a= -3/r



S ∞ = $\frac{a}{1-r}$ = 25/2

$\frac{-3}{r}$ x $\frac{1}{1-r}$ = 25/2;

25r - 25r$^2$ = -6;

25r$^2$ - 25r - 6 = 0

(5r +1) (5r - 6) = 0

r= -1/5 and 6/5

10.

P and Q are two linear transformations in the X-Y plane defined by

P: (x, y) → (-3x + 6y, 4x + y) and

Q: (x, y) → (2x-3y, -4x - 6y).

(a) Write down the matrices of P and Q. (b) What is the image of (-2,-3) under the transformation Q?

(c) Obtain a single transformation representing the transformation Q followed by P.

(d) Find the image of (1,4) when transformed by Q followed by P.

(e) Find the image $P^1$ of the point (-√2,2√2) under an anticlockwise rotation of 225° about the origin.

SOLUTION

P: (x, y) → (2x+3y, 3x - y)

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -3x & 6y \\ 4x & +y \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 6 \\ 4 & +1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$P = \begin{pmatrix} -3 & 6 \\ 4 & +1 \end{pmatrix}$$



Q: (x, y) → (2x-3y, -4 - 6y)

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2x & -3y \\ -4x & -6y \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -3 \\ -4 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$Q = \begin{pmatrix} 2 & -3 \\ -4 & -6 \end{pmatrix}$$

(b) $\begin{pmatrix} 2 & -3 \\ -4 & -6 \end{pmatrix} \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

= $\begin{pmatrix} 2[-2] & -3[-3] \\ -4[-2] & -6[-3] \end{pmatrix}$

= $\begin{pmatrix} -4 & +9 \\ 8 & 18 \end{pmatrix}$

= $\begin{pmatrix} 5 \\ 26 \end{pmatrix}$

: The image of (-2,-3) is (5,26)

(c) PQ = $\begin{pmatrix} -3 & 6 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ -4 & -6 \end{pmatrix}$

$\begin{pmatrix} -3[2] + 6[-4] & -3[-3] + 6[-6] \\ 4[2] + 1[-4] & 4[-3] + 1[-6] \end{pmatrix}$

= $\begin{pmatrix} -6 - 24 & 9 - 36 \\ 8 - 4 & -12 - 6 \end{pmatrix}$ = $\begin{pmatrix} -30 & -27 \\ 4 & -18 \end{pmatrix}$

= PQ: (x,y) → (-30x - 27y, 4x - 18y)

(d) $\begin{pmatrix} -30 & -27 \\ 4 & -18 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

= $\begin{pmatrix} -30[1] & +[-27][4] \\ 4[1] & +[-18][4] \end{pmatrix}$

= $\begin{pmatrix} -30 & -108 \\ 4 & -72 \end{pmatrix}$ = $\begin{pmatrix} -138 \\ -68 \end{pmatrix}$

: Image of the point (1,4) is (-138,-68)

(e) T = $\begin{pmatrix} cos225^\circ & -sin225 \\ sin225^\circ & cos225^\circ \end{pmatrix}$

= $\begin{pmatrix} -\sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & -\sqrt{2}/2 \end{pmatrix} \begin{pmatrix} -\sqrt{2} \\ 2\sqrt{2} \end{pmatrix}$

= $\begin{pmatrix} 1 & +2 \\ 1 & -2 \end{pmatrix}$ - $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

: The image of $P^1$ is (3,-1)

11.

(a) Find the equation of the normal to the curve y = $(x^2 - x + 1)(x - 2)$ at the point where the curve cuts the X - axis.

(b) The coordinates of the pints P, Q and R are (-1, 2), (5, 1) and (3, -4) respectively. Find the equation of the line joining Q and the midpoint of line PR.

SOLUTION

(a) From $y = (x^2 - x + 1)(x - 2) - x + 1)(x - 2)$

$= x^3 - 3x^2 + 3x - 2$

$dy/dx = 3x^2 - 6x + 3$

$= x^2 - 2x + 1$

$(x - 1) = 0$ twice ; $x = 1$

$y = (1^2 - 1 + 1)(1-2)$ ; $y = -1$

Gradient $m_1$ of tangent = $y/x = -1/1 = -1$

Using $m_1 m_2 = -1$

Gradient, $m_2$ of normal =1

Using $y-y_1 = m_2(x - x_1)$

$y + 1 = x - 1$; $x - y - 2 = 0$

$x_1, y_1 = (-1, 2), , x_2, y_2, =(3, -4)$

Mid point of PR = $\frac{3-1, -4+2}{2,2}$

= (1-1)

Using $\frac{y_2-y_1}{x_2-x_1} = \frac{y-y_1}{x-x_1}$

$\frac{1--1}{5-1} = \frac{y-1}{x-5}$;

$\frac{2}{4} = \frac{y-1}{x-5}$

$2(y-1) = x - 5$; $2y - 2 = x - 5$

$2y - x = -3$

$x - 2y - 3 = 0$

12.

A box contains 5 red, 7 blue and 4 green identical bulbs. Two bulbs are picked at random from the box without replacement.

Calculate the probability of picking: (a) same color of bulbs; (6) different color of bulbs (c) at least one red bulb.

. Total bulbs: n(Red)=5, n(B) = 7, n(G) = 4

(5+7+4) = 16

p(R)= 5/16, p(B) = 7/16, p(G) = 4/16

(a) p(All same colour of bulbs) = p(RR) Or p(BB) or p(GG)

$\frac{5}{16} * \frac{4}{15} + \frac{7}{16} * \frac{6}{15} + \frac{4}{16} * \frac{3}{15}$

$= \frac{1}{12} + \frac{7}{40} + \frac{1}{20}$

$= \frac{10+21+6}{120} = \frac{37}{120}$

(b) All different colors = 1-p (All the same colour) =

$1 - \frac{37}{120} = \frac{83}{120}$

(c) p(At least one red) = p(RR) + p(RB) + p(RG)

$\frac{5}{16} * \frac{4}{15} + \frac{5}{16} * \frac{7}{15} + \frac{5}{16} * \frac{4}{15}$

$= \frac{1}{12} + \frac{7}{48} + \frac{1}{12}$

$= \frac{8+7}{48} = \frac{15}{48}$

$= \frac{5}{16}$



13.

The table shows the frequency distribution of heights (in cm) of pupils in a certain school.

| Heights | 100-109 | 110-119 | 120-129 | 130-139 | 140-149 | 150-159 | 160-169 |
|---|---|---|---|---|---|---|---|
| Frequency | 27 | 58 | 130 | 105 | 50 | 25 | 5 |

(a) (i) Construct a cumulative frequency table.

(ii) Use the table to draw a cumulative frequency curve.

(b) Using the curve, estimate the:

(i)median height;

(ii) inter quartile range

(iii) percentage of students whose heights are most 130cm.

SOLUTION

| Height (in cm) | Frequency |
|---|---|
| Not exceeding 99.5 | 0 |
| Not exceeding 109.5 | 27 |
| Not exceeding 119.5 | 85 |
| Not exceeding 129.5 | 215 |
| Not exceeding 139.5 | 320 |
| Not exceeding 149.5 | 370 |
| Not exceeding 159.5 | 395 |
| Not exceeding 169.5 | 400 |

(ii)

bi) Median 1/2 of N = 1/2 x 400 = 200

Cumulative frequency level From the curve, it is 118.5cm

(ii) Inter quartile range = upper quartile - lower quartile upper quartile

= 3/4 of N

= 3/4 x 400 = 300

Cumulative frequency level From the curve, it is 126.5cm

lower quartile = 1/4 of N

= 1/4 x 400 = 100

Cumulative frequency level From the curve, it is 111.0cm

Inter quartile range = 126.5cm -111.0cm = 15.5cm

(ii) From the curve, students whose heights are at most 130cm are 240.

Percentage óf students whose heights are at most 130cm = $\frac{240}{400}$ x 100

= 60%

14.

The position vectors of P, Q and R with respect to the origin are (4i-5j), (i+3j) and (-5i+2j) respectively. If PQRM is a parallelogram, find:

(a) the coordinates of M;
(b) the acute angle between P¯M and P¯Q, correct to the nearest degree.
SOLUTION

$\overline{PQ} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$

$\overline{MR} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5 & -x \\ 2 & -y \end{pmatrix}$

$\begin{pmatrix} -3 \\ 8 \end{pmatrix} = \begin{pmatrix} -5 & -x \\ 2 & -y \end{pmatrix}$

⇒ -5 - x = -3; x = -2

⇒ 2 - y = 8; y = -6
The coordinate of M (-2,-6)

(b) $\overline{PM} = \overline{OM} - \overline{OP}$

$= \begin{pmatrix} -2 \\ -6 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \end{pmatrix}$



$\overline{PQ} = \overline{OQ} - \overline{OP}$

$= \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$

$|\overline{PM}| = \sqrt{(-6^2 + [-1^2])} = \sqrt{37}$

$|\overline{PQ}| = \sqrt{(-3^2 + 8^2)} = \sqrt{73}$

$\cos Ø = \frac{PM.PQ}{|\overline{PM}||\overline{PQ}|}$

$\cos Ø = \frac{[-6i-j].[-3i+8j]}{\sqrt{37}.\sqrt{73}} = \frac{10}{\sqrt{2710}}$

$\Rightarrow Ø = \cos^{-1} \frac{10}{\sqrt{2710}}$

: Ø = 78.91° ≈ 79°

15.

(a) A girl threw a stone horizontally with a velocity of 30m/s from the top of a cliff 50m high. How far from the foot of the cliff does the stone strike the ground? [Take g= 10m/s$^2$]

SOLUTION

(a) Time taken to reach the ground

Using $S = ut + 1/2gt^2$

(u= 0 for a body falling from rest)

$50 = 0 \times t + 1/2 \times 10t^2$

$5t^2 = 50;$

$t^2 = 50/5 = 10$

$t = \sqrt{10} = 3.16$

Distance when it strikes the ground This journey is independent of gravity

d =vt =30 x 3.16 = 94.8m



(b) A body A, of mass 2kg is held in equilibrium by means of two strings AP and AR. AP is inclined at 56° to the upward vertical and AR is horizontal.

Find the tensions $T_1$, and $T_2$, in the strings [Take g= 10ms$^2$]

SOLUTION

(b) Using Lami's rule,

$$\frac{20}{sin34} = \frac{T_1}{sin90} = \frac{T_2}{sin56}$$

$$= T_1 \frac{20.sin90}{sin34} = T_2 \frac{20.sin56}{sin34}$$

$T_1 = 35.79N$ and $T_2 = 29.65N$